

# III Encontro CEA FEL

Meeting on Functional Analysis, Linear Structures and Applications

5-6 September - Universidade do Algarve

## List of invited speakers

*Olga Azenhas (Universidade de Coimbra)*  
*Luis Castro (Universidade de Aveiro)*  
*Susana Furtado (Universidade do Porto)*  
*Oleksiy Karlovykh (Universidade Nova de Lisboa)*  
*Jocelyn Lochon (Universidade de Lisboa)*  
*Nenad Manojlović (Universidade do Algarve)*

**FCT**  
Fundação para a Ciência e a Tecnologia  
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

 **UAlg FCT**  
UNIVERSIDADE DO ALGARVE  
FACULDADE DE CIÊNCIAS E TECNOLOGIA

**C E A F E L**

## Organizers:

Ana Conceição (Universidade do Algarve); Ângela Mestre (Universidade de Lisboa);  
Juan Rodríguez (Universidade do Algarve); Rui Marreiros (Universidade do Algarve).

# III Encontro CEA FEL

## Meeting on Functional Analysis, Linear Structures and Applications

**5-6 September 2017, Universidade do Algarve**

The Research Center for Functional Analysis, Linear Structures and Applications, CEA FEL, is pleased to announce the *III Encontro CEA FEL - Meeting on Functional Analysis, Linear Structures and Applications* to be held at Universidade do Algarve, Portugal, on 5-6 September 2017.

This meeting aims to develop interactions between mathematicians working in Functional Analysis, Group Representation Theory, Matrix Theory and Linear Systems, and other relevant areas. Participation is open to anyone with interests in these and other related areas.

This 2-day Workshop will be held at the Universidade do Algarve on 5-6 September 2017. The programme will consist of 45 minutes talks presented by invited speakers. On the afternoon of Wednesday, 6th September, there will be a round table restricted to members and collaborators of CEA FEL.

### *List of invited speakers*

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Luís Castro (Universidade de Aveiro)  
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Nenad Manojlović (Universidade do Algarve)

### *Organizers*

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### *Support*

CEA FEL: Centro de Análise Funcional, Estruturas Lineares e Aplicações  
FCT-UAlg: Faculdade de Ciências e Tecnologia da Universidade do Algarve  
FCT: Fundação para a Ciência e a Tecnologia  
projeto UID/MAT/04721/2013



C E A F E L

# Program<sup>1</sup>

Tuesday, September 5

14:15 – 14:30 Opening Session

14:30 – 15:20 On the representation theory of the Infinite Unitriangular group

Jocelyn Lochon (Universidade de Lisboa)

Abstract: Let  $U(q)$  denote the set of infinite nilpotent matrices over the finite field with  $q$  elements, the group  $1 + U(q)$  is called the infinite unitriangular group. It is a wild group (not type I), thus the irreducible representations do not admit any reasonable parametrization.

Nevertheless the set of all characters (positive definite, normalized class functions) characterizes up to quasi-equivalence the factor representations of type I and II. With the pointwise convergence topology the characters form a Choquet's simplex hence fully described by its extreme/indecomposable points.

The goal of this talk is twofold: to define what is meant by a Supercharacter theory of  $1 + U(q)$  and to construct and describe a family of supercharacter theories. As a by-product we present a description of the extreme characters of  $1 + U(q)$ .

15:20 – 16:10 Semi-Fredholm theory for singular integral operators with shifts and slowly oscillating data

Oleksiy Karlovych (Universidade Nova de Lisboa)

Abstract: Let  $\alpha, \beta$  be orientation-preserving homeomorphisms of  $[0, \infty]$  onto itself, which have only two fixed points at 0 and  $\infty$ , and whose restrictions to  $\mathbb{R}_+ = (0, \infty)$  are diffeomorphisms, and let  $U_\alpha, U_\beta$  be the corresponding isometric shift operators on the space  $L^p(\mathbb{R}_+)$ ,  $1 < p < \infty$ , given by  $U_\mu f = (\mu')^{1/p} (f \circ \mu)$  for  $\mu \in \{\alpha, \beta\}$ . We will discuss criteria for the left/right Fredholmness and  $n$ -normality/ $d$ -normality of singular integral operators of the form

$$A_+ P_\gamma^+ + A_- P_\gamma^-,$$

where

$$A_+ = \sum_{k \in \mathbb{Z}} a_k U_\alpha^k, A_- = \sum_{k \in \mathbb{Z}} b_k U_\beta^k$$

are operators in the Wiener algebras of functional operators with shifts and the operators  $P_\gamma^\pm = (I \pm S_\gamma)/2$  are associated to the weighted Cauchy singular integral operator

$$(S_\gamma f)(t) = \frac{1}{\pi i} \int_{\mathbb{R}_+} \left( \frac{t}{\tau} \right)^\gamma \frac{f(\tau)}{\tau - t} d\tau,$$

where  $\gamma \in \mathbb{C}$ , satisfies  $0 < \frac{1}{p} + \operatorname{Re} \gamma < 1$ . We assume that the coefficients  $a_k, b_k$  for  $k \in \mathbb{Z}$  and the derivatives of the shifts  $\alpha', \beta'$  are bounded continuous functions on  $\mathbb{R}_+$  which may have slowly oscillating discontinuities at 0 and  $\infty$ .

This talk is based on the joint work with Yuri Karlovich (Cuernavaca, Mexico) and Amarino Lebre (Lisbon, Portugal).

16:10 – 16:40 Coffee Break

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<sup>1</sup> The meeting will be held in room C8 3.24.

## 16:40 – 17:30 Wave diffraction involving a crack with different face impedances

Luís Castro (Universidade de Aveiro)

**Abstract:** The talk will be devoted to study classes of plane wave diffraction problems by a region which involves a crack with different impedance boundary conditions. Conditions on the wave number and impedance parameters are found to ensure the well-posedness of the problems in a scale of Bessel potential spaces. Under such conditions, representations of the solutions are also obtained upon the consideration of some associated operators which will combine operators of Wiener-Hopf and Hankel type. The talk is based on joint work with D. Kapanadze.

## 17:30 – 18:00 Bruhat Order for Symmetric $(0,1)$ -Matrices

Susana Furtado (Universidade do Porto)

**Abstract:** Extending the Bruhat order for permutation matrices, in [1] a Bruhat order for the class of  $m$ -by- $n$   $(0,1)$ -matrices with prescribed row and column sum vectors was defined. Minimal matrices for this Bruhat order (a partial order) were studied in this paper and in the subsequent paper [2]. In this talk we present some results, obtained in [3], related with the description of the minimal matrices in the Bruhat order for the class of symmetric  $(0,1)$ -matrices with given row sum vector. We start by giving some properties of these minimal matrices. We also present minimal matrices for the Bruhat order on some particular such classes of symmetric  $(0,1)$ -matrices. Some connections with the term rank of a matrix will be established.

### References

- [1] R.A. Brualdi and S.-G. Hwang, *A Bruhat order for the class of  $(0,1)$ -matrices with row sum vector  $R$  and column sum vector  $S$* , *Electronic Journal of Linear Algebra* 12 (2004), 6-16.
- [2] R.A. Brualdi and L. Deaett, *More on the Bruhat order for  $(0,1)$ -matrices*, *Linear algebra and its applications* 421 (2007), 219-232.
- [3] H. F. Cruz, R. Fernandes, S. Furtado, *Minimal matrices in the Bruhat order for symmetric  $(0,1)$ -matrices*, *Linear Algebra and its Applications* 530 (2017), 160-184.

Wednesday, September 6

## 10:30 – 11:20 Generalized Gaudin Algebras

Nenad Manojlović (Universidade do Algarve)

**Abstract:** A reflection matrix and the so-called classical reflection equation are a way to introduce non-periodic boundary conditions compatible with complete integrability of the system. The linear bracket for the modified Lax matrix can be defined using the corresponding non-unitary classical r-matrix. This bracket is by definition anti-symmetric and it also obeys the Jacobi identity. Therefore, the entries of the modified Lax matrix generate an infinite dimensional Lie algebra, the generalized Gaudin algebra. An important problem is the classification of the generalized Gaudin algebras, or equivalently the classification of the non-unitary classical r-matrices. As an illustration, starting from  $\mathfrak{sl}(2)$ -invariant unitary rational r-matrix and the corresponding classical K-matrix we will produce new examples by the methods of fusion and twisting.

## 11:20 – 12:10 Skew RSK and coincidence of LR commutators

Olga Azenhas (Universidade de Coimbra)

**Abstract:** Sagan and Stanley have given a skew-version of RSK. Given  $\mu \subseteq \alpha, \beta$  three partitions, a special case, is a bijection between pairs of skew-tableaux  $(T, U) \rightarrow (P, Q)$  with shapes  $(\alpha/\mu, \beta/\mu)$  and  $(\lambda/\beta, \lambda/\alpha)$ , so that  $P, T$  are Knuth equivalent, and  $Q, U$  are of the same weight. The correspondence is

based on an internal insertion procedure which requires a sequence of inner corners in  $T$  encoded by the skew tableau  $U$ .

It is shown that the word listing the row indices of the entries of  $U$ , in the standard order, is such that Knuth equivalence provides a sufficient condition for the equality of  $P$ -tableaux. The Knuth commutation satisfied by internal insertion operations is instrumental to prove the coincidence between the Benkart-Sottile-Stroomer switching map and an LR commuter by the author. It then follows that all LR commuters, known up to date, are involutions and coincide. This completely fulfills the question raised by Pak and Vallejo on the coincidence of LR commuters.

12:10 – 14:00 Lunch

14:00 – 16:30 Round Table

- **Symbols for non local  $C^*$  algebras**  
Amélia Bastos (Instituto Superior Técnico)  
Abstract: Symbols for  $C^*$  algebras of bounded linear operators on  $L^2$  which are generated by unitary representations of groups of diffeomorphisms, multiplication operators by piecewise quasicontinuous functions and by the Cauchy singular integral operator, are presented. Faithful representations for the associated quotient  $C^*$  algebras are established by using spectral measures and the local trajectory method for  $C^*$  algebras associated with  $C^*$  dynamical systems. Open problems are discussed having in mind previous results for  $C^*$  algebras with piecewise slowly oscillating coefficients.
- **Factorization of almost periodic matrix functions**  
António Bravo (Instituto Superior Técnico)  
Abstract: Results of factorization of almost periodic  $2 \times 2$  matrix functions are indicated. Some factorization results for  $n \times n$  matrix functions are presented having in mind the application to convolution-type operators on a union of intervals.
- **Shintani descent for algebra groups**  
Carlos André (Universidade de Lisboa)  
Abstract: We briefly explain trace maps between characters of algebra groups over finite fields, and discuss certain asymptotic properties related to these maps.
- **Fredholm criteria and representations of  $C^*$ -algebras**  
Catarina Carvalho (Universidade de Lisboa)  
Abstract: Several classes of (pseudo)differential operators on domains with boundary can be modelled by operators on groupoids. We explain how one can relate Fredholm criteria on these spaces with the representation theory of the associated groupoid  $C^*$ -algebras, using strictly spectral and exhausting families.
- **Approximation sequences and their spectral properties**  
Helena Mascarenhas (Instituto Superior Técnico)  
Abstract: Finite sections of a large class of convolution type operators and their properties are presented. Behind the results are local principles and variations of the Lifting theorem.
- **Combinatorics of a Generalized Narayana Identity**  
José Agapito Ruiz (Universidade de Lisboa)  
Abstract: The Narayana identity is a well-known formula that expresses the classical Catalan numbers as sums of the ordinary Narayana numbers. In this talk I will present a generalization of the Narayana identity for a family of Riordan arrays that includes the Aigner array, the Shapiro array, and several other generalized Catalan arrays recently studied. I will give a combinatorial description for such an identity based on non-crossing partitions. This talk is based on joint work with Ângela Mestre, Pasquale Petrullo and Maria M. Torres.

- **The kernel of singular integral operators with a finite group of linear-fractional shifts**  
 Juan Rodríguez (Universidade do Algarve)  
Abstract: Let  $T$  denote the unit circle. In our talk we will consider a singular integral operators with two shifts of linear-fractional type, acting in the Lebesgue space  $L_p(T)$ ,  $p \in (1, \infty)$ . We suppose that one shift preserves the orientation of  $T$ , the other changes it and, in addition, it is assumed that these shifts generate a finite group. We show that the original singular integral operator with two shifts is, after extension, equivalent to a singular integral operator without shifts and with square matrix coefficients, the order of which depends on the order of the shifts under consideration. By exploiting the properties of the factorization of the symbol of this last operator, it is possible to characterize the kernel of the original operator.
- **Symmetry classes of tensors and Specht modules**  
 Maria Manuel Torres (Universidade de Lisboa)  
Abstract: We construct induced bases for critical orbital spaces indexed by standard tableaux. As a corollary we obtain a realization of the complete set of irreducible complex representations of the symmetric group in terms of decomposable symmetrized tensors (joint work with José Perdigão Dias da Silva, Fátima Rodrigues and Pedro C. Silva).
- **Isomonodromic deformations and the Schlesinger system**  
 Nuno António (Instituto Superior Técnico)  
Abstract: Given a linear first order matrix ODE, with rational coefficients, the position of its singularities, in the complex plane, are changed in order to keep the corresponding monodromy data unchanged. This variation defines an integrable hamiltonian system, known as the Schlesinger system, for the residues as functions of the positions of the poles.
- **An algorithm for the factorization of some classes of matrix functions**  
 Paulo Semião e Juan Rodríguez (Universidade do Algarve)  
Abstract: In this work we present a computer algorithm for the explicit factorization, on the unit circle, of some classes of matrix functions. We have been working together in an efficient implementation of a software tool that not only accomplishes the factorization of rational matrices, but could also be applied to a more general class of matrices. We will show some examples of computations for the explicit factorization of matrices.

## Posters

- **PDLs and Flowcharts in Operator Theory**  
 Ana C. Conceição (Universidade do Algarve)  
Abstract: In recent years, several software applications were made available to the general public with extensive capabilities of symbolic computation. These applications, known as computer algebra systems (CAS), allow to delegate to a computer all, or a significant part, of the symbolic calculations present in many mathematical algorithms. The main goal of this work is to present some Program Design Languages (PDLs) and flowcharts developed by us, and others, within Operator Theory.  
**References**  
 A.C. Conceição, V.G. Kravchenko, J.C. Pereira (2013). Computing some classes of Cauchy type singular integrals with Mathematica software. *Adv.Comput.Math.* 39(2), 273-288. Springer US  
 A.C. Conceição, R.C. Marreiros, J.C. Pereira (2016). Symbolic computation applied to the study of the kernel of a singular integral operator with non-Carleman shift and conjugation. *Math.Comput.Sci.* 10(3), 365-386. Springer International Publishing  
 A.C. Conceição, J.C. Pereira (2016). Exploring the spectra of some classes of singular integral operators with symbolic computation. *Math.Comput.Sci.* 10(2), 291-309. Springer International Publishing

- On the kernel of a singular integral operator with a non-Carleman shift and conjugation

Rui Marreiros (Universidade do Algarve)

Abstract: Let  $\Gamma$  be the unit circle or the one-point compactification of the real line. On the Hilbert space  $L_2(\Gamma)$ , the singular integral operator with non-Carleman shift and conjugation

$K = P_+ + (aI + AC)P_-$  is considered, where  $P_{\pm}$  are the Cauchy projectors,  $A = \sum_{j=0}^m a_j U^j$ ,

$a, a_j, j = \overline{1, m}$ , are continuous functions on  $\Gamma$ ,  $U$  is the shift operator and  $C$  is the operator of complex conjugation. Some estimates for the dimension of the kernel of the operator  $K$  are obtained.